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# SOLAR OPTICAL PROPERTIES OF WINDOWS

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## SUMMARY

To properly calculate dynamic solar gain in buildings, one must know the optical properties of the window in detail. In this paper, we develop a complete set of calculation procedures for determining the solar transmittance, reflectance, and absorptance of a window composed of an arbitrary number of partially transparent layers. Any layer may have a thin-film multilayer coating, such as an anti-reflection coating for increasing solar transmittance, a solar control film for reducing solar heat gain, or a transparent heat-reflecting mirror for improving thermal resistance. The results of sample calculations are given over the range of incidence angles for conventional and advanced energy conserving window designs.

KEY WORDS Windows Thin-film Optics

## INTRODUCTION

Solar heat gain through a window system can be determined from the overall transmittance and the absorptance of each pane or layer as a function of the angle of incidence. The transmittance value is required to determine the fraction of direct solar radiation entering the room for any given angle of incidence, $\theta$, which changes with the sun's motion during the day. A hemispherical average transmittance value is needed to account for diffuse sky radiation and reflected ground radiation. The absorptance is needed because some of the short-wave solar radiation absorbed by the window is reradiated and convected to the room. In a window composed of layers separated by large thermal resistances, e.g. air spaces, the absorptance must be known for each layer, since a different fraction of the absorbed solar energy will be transferred inwards depending on the resistances between layers and their respective temperatures.

Triple-glazed windows are widely used in northern climates and quadruple-glazed windows are available on a smaller scale. To assess the relative merits of triple and quadruple glazing requires that the net annual energy performance of these window systems be carefully determined in different climates and orientations. Of particular interest in terms of their energy performance is that the balance between reducing heat loss and losing transmitted solar energy be evaluated. Another option is the addition of transparent plastic layers in the air space of double-glazed windows to reduce convective heat transfer or, alternatively, to use as a retrofit product for existing single-glazed windows (Selkowitz, 1979). Already in widespread use are surface coatings such as solar control films to reduce unwanted solar heat gain; transparent heat mirror coatings that add thermal insulation are now on the market in Europe and will be available in the United States in the near future. Because of the high cost and subtle performance differences of some of these window systems, accurate evaluation of their potential for conserving energy is especially important.

A commonly used set of algorithms for calculating heat transfer in buildings is that developed by ASHRAE (Lokmanhekim, 1975). One of these algorithms, 'TAR', allows the calculation of transmission, absorption, and reflection factors for single-pane or double-pane windows only. The procedure used in 'TAR' is based on the work of Mitalas and Stephenson (1962). The first step is to calculate the Fresnel reflection coefficients at the air–glazing interfaces. Closed-form expressions for total transmission ($T$) and individual absorptions of the two panes ($A_1$, $A_2$) are obtained from these single interface reflectances

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and the specified bulk absorption coefficient of the glazing. This information is expressed in condensed form for clear and heat-absorbing glass windows; i.e. a least-squares fit of $T(\theta)$, $A_1(\theta)$, and $A_2(\theta)$ to a polynomial in $\cos\theta$ (Lokmanhekim, 1975; Mitalas and Stephenson, 1962; ASHRAE, 1977). These coefficients are then used in a subroutine that calculates hourly heat transfer.

The algorithms presented in this paper, by using recursion methods rather than closed form equations, extend the capabilities of the ASHRAE model to more than two layers. Thin-film multilayer coatings are treated in a fashion that is compatible with the bulk layer approach. This procedure will allow building energy analysis programs to simulate the effect of advanced energy-conserving window systems (described above) on annual energy performance. Results are given for windows made from glass, polyester film, and high-transmission polyester with an anti-reflection coating.

## SINGLE INTERFACE OPTICS

At the boundary between two semi-infinite media there are no multiple reflections and no absorption. Reflectance ($R$) and transmittance ($T$) are given by Fresnel's equations. In the following equations, $n_i$ is the index of refraction of medium $i$; $\theta_i$ is the angle between the direction of propagation and the normal to the boundary in medium $i$. The subscripts TE and TM refer to transverse electric and transverse magnetic polarizations, respectively.

$$R_{\mathrm{TE}} = \left[\frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}\right]^2 \tag{1}$$

$$T_{\mathrm{TE}} = 1 - R_{\mathrm{TE}} \tag{2}$$

$$R_{\mathrm{TM}} = \left[\frac{n_1/\cos\theta_1 - n_2/\cos\theta_2}{n_1/\cos\theta_1 + n_2/\cos\theta_2}\right]^2 \tag{3}$$

$$T_{\mathrm{TM}} = 1 - R_{\mathrm{TM}} \tag{4}$$

The angle of incidence is given and the angle of refraction is determined from

$$\cos\theta_2 = [1 - (n_1/n_2)^2 \sin^2\theta_1]^{1/2} \tag{5}$$

The functional dependence on polarization can be eliminated in equations (1) and (3) by defining an effective index of refraction for each polarization, i.e.

$$n \equiv \begin{cases} n \cos\theta & \text{for TE polarization} \\ n/\cos\theta & \text{for TM polarization} \end{cases} \tag{6}$$

Throughout the rest of this paper these conventions will be used in order to treat polarization in a completely general manner. For an arbitrarily plane-polarized wave, the reflectance is given by

$$R = R_{\mathrm{TM}} \cos^2\phi + R_{\mathrm{TE}} \sin^2\phi \tag{7}$$

where $\phi$ is the angle of polarization with respect to the plane of incidence. For 'natural' unpolarized light, $\phi$ fluctuates in a random fashion, and only the time average $\langle R\rangle$ is measurable;

$$\langle R\rangle = \tfrac{1}{2}(R_{\mathrm{TM}} + R_{\mathrm{TE}}) \tag{8}$$

The average transmittance is given by a similar expression:

$$\langle T\rangle = \tfrac{1}{2}(T_{\mathrm{TM}} + T_{\mathrm{TE}}) \tag{9}$$

The dispersion of the index of refraction implies a wavelength dependence of $R$ and $T$ which does not appear explicitly until we consider thin film interference effects. For most clear glazing materials the optical properties are sufficiently constant over the solar spectrum that the use of band averaged optical constants does not introduce significant error.

## SINGLE LAYER ABSORPTION

Absorption in the layers is taken into account by introducing an absorption coefficient $k[\text{cm}^{-1}]$, which may be derived from a single normal transmittance measurement and the index of refraction, as follows: Reflectance at either interface of a glazing material sample measured in air ($n_1 = 1$) at normal incidence is derived from equation (1) or equation (3),

$$\rho = \left[\frac{1-n}{1+n}\right]^2 \tag{10}$$

For a given thickness ($h$), the absorption on a single pass within the layer for any angle of refraction is given by Bouger's Law:

$$a = 1 - \exp(-kh/\cos\theta) \tag{11}$$

Summing all the rays which appear on the right in Figure 1 gives the normal transmittance ($T_n$), in terms of $a_n$,

$$\begin{aligned} T_n &= (1-\rho)^2(1-a_n)\left[1+\sum_{i=1}^{\infty}[\rho^2(1-a_n)^2]^i\right] \\ &= \frac{(1-\rho_n)^2(1-a_n)}{1-\rho_n^2(1-a_n)^2} \end{aligned} \tag{12}$$

Solving equation (12) for $a_n$ yields

$$a_n = 1 + \frac{(1-\rho)^2 - [(1-\rho)^4 + 4\rho^2 T_n^2]^{1/2}}{2\rho^2 T_n} \tag{13}$$

where the negative root was chosen to make $a_n$ less than 1. From equation (11) with $\theta = 0$,

$$k = \frac{1}{h}\ln(1-a_n) \tag{14}$$

Absorption at any angle can now be obtained from equation (11).

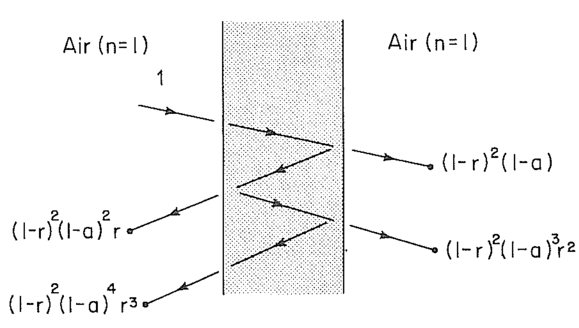


Figure 1. Multiple reflections in a single absorbing layer. Reflectance at either interface is given by $r$ and absorptance on a single pass through the material is given by $a$

## SERIES OF THICK LAYERS

Consider a collection of plane parallel layers whose thicknesses are large compared with a wavelength of incident sunlight. For most windows the materials will be glass and plastic sheets separated by air or other gases. The most general situation is shown in Figure 2: $s$ layers including two semi-infinite air layers on either side, i.e. $s-1$ interfaces.

$R_{i,j}$ and $T_{i,j}$ refer to reflection from and transmission through that portion of the window bounded by and including boundaries $i$ and $j$, as if the segment were standing alone (i.e. letting layers $i$ and $j+1$ become semi-infinite). Summing multiple reflections gives, for $i<j$,

$$T_{i,j}=\frac{T_{i,j-1}T_{j,j}(1-a_j)}{1-R_{j,j}R_{j-1,i}(1-a_j)^2} \tag{15}$$

and

$$R_{i,j}=R_{i,j-1}+\frac{T_{i,j-1}^2R_{j,j}(1-a_j)^2}{1-R_{j,j}R_{j-1,i}(1-a_j)^2} \tag{16}$$

Reversing the direction of the beam shows that transmittance is identical from either side,

$$T_{j,i}=T_{i,j} \tag{17}$$

and the back reflectance is

$$R_{j,i}=R_{j,j}+\frac{T_{i,j}^2R_{j-1,i}(1-a_j)^2}{1-R_{j-1,i}R_{j,j}(1-a_j)^2} \tag{18}$$

Starting from the incident side and proceeding to the right in Figure 2 until the layers are exhausted will give the overall transmittance and the reflectances from front and back; $T=T_{1,s}$, $R^{\mathrm{f}}=R_{1,s}$ and $R^{\mathrm{b}}=R_{s,1}$, respectively. If there is only one absorbing layer or if all the absorbing layers are connected by small thermal resistances, then the only further quantity to be determined is the total absorptance for radiation incident from the front side, $A_{\mathrm{t}}=1-R^{\mathrm{f}}-T$. In general, it is necessary to find the absorptance in each layer, such as in the two panes of an insulated window unit. Figure 3 shows layer $j$ as that portion of

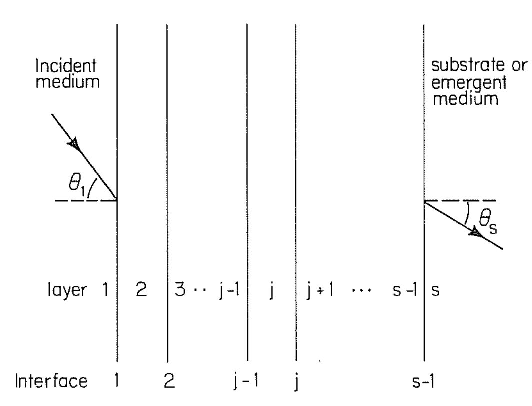


Figure 2. General structure of bulk layers in a window, or thin-film layers on a bulk substrate. Each layer is counted, including air. Layer 1 is always air (the atmosphere) and layer $s$ may be either air or a thin film substrate

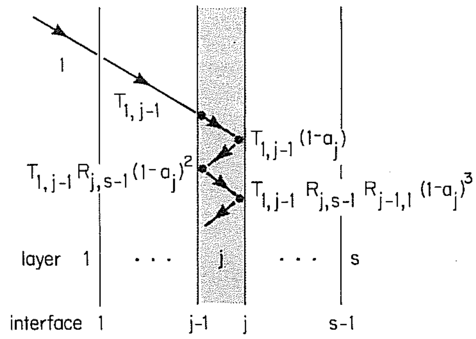


Figure 3. Absorption in a bulk window layer counting absorption due to multiple reflections from surrounding layers. $T_{i,j}$ is the transmittance through interfaces $i$ to $j$; $R_{i,j}$ is the reflectance from interfaces $i$ to $j$. The absorptance in layer $j$ is given by $a_j$

the window for which absorption is to be calculated. Once again, summing multiple reflections gives the fraction of sunlight absorbed in layer $j$, for $1<j<s$.

$$A_j = T_{i,j-1}a_j\left[\frac{1+(1-a_j)R_{j,s-1}}{1-(1-a_j)^2R_{j,s-1}R_{j-1,1})}\right] \tag{19}$$

## THIN-FILM MULTILAYER MODEL

The calculation of multiple-beam interference in thin-film stacks must be given special attention. Figure 2 represents the situation just as well for thin films as for bulk materials but, as the thickness of the layers approaches the wavelength range of solar light, interference effects become important. In such a case, layer 1 is either the atmosphere or a thick gas layer between panes of glass. The gas may not be air but, when considering its interaction with solar radiation, it can be treated as if it were. The purpose of using a gas other than air would be to modify the thermal infrared or convective heat transfer only. Layer $s$ is necessarily a solid substrate.

In principle the glass or plastic substrate could be treated as just another layer in which interference occurs, but the assumption of parallel sidedness breaks down due to the large fluctuations in thickness with respect to wavelength in a piece of commercial glass. Treating the substrate like the thin layers would require solving the electromagnetic field equations with random boundary conditions; short of this, we simply treat the light in thick layers incoherently, and treat interference effects only in thin layers.

The electromagnetic boundary value problem is solved by Heavens (1960), where it is shown that the tangential components of the electric and magnetic fields in successive layers are related by

$$\begin{bmatrix} E_j \\ H_j \end{bmatrix} = \begin{bmatrix} \cos\delta_{j+1} & \mathrm{i}\sin\delta_{j+1}/n_{j+1} \\ \mathrm{i}n_{j+1}\sin\delta_{j+1} & \cos\delta_{j+1} \end{bmatrix}\begin{bmatrix} E_{j+1} \\ H_{j+1} \end{bmatrix}$$

$$= [M_{j+1}]\begin{bmatrix} E_{j+1} \\ H_{j+1} \end{bmatrix} \tag{20}$$

where the phase retardation across the $j$th layer is

$$\delta_j = \frac{2\pi n_j h_j}{\lambda}\cos\theta_j \tag{21}$$

Recalling the definitions in equation (6), this result is valid for either polarization. It is possible for a highly absorbing material to be a part of a transparent window if the layer is thin enough. In this case, $n$ and $\theta$ are complex; $n \equiv n - \mathrm{i}k$, where $k$ is the extinction coefficient.

Repetition of this process gives us a relation between the fields in the incident medium (1) and the last film in the stack $(s-1)$,

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = [M] \begin{bmatrix} E_{s-1} \\ H_{s-1} \end{bmatrix} \tag{22}$$

where

$$[M] = [M_2][M_3] \cdots [M_{s-2}][M_{s-1}] \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{23}$$

Fields representing the rightward travelling portion of the wave are denoted with a '+' superscript and the leftward travelling part with a '−' superscript, so that $E_j = E_j^+ + E_j^-$. There is no reflection in the semi-infinite substrate; thus, there is only a rightward-going wave which gives the boundary conditions at the final $(s-1, s)$ interface,

$$\begin{bmatrix} E_{s-1} \\ H_{s-1} \end{bmatrix} = \begin{bmatrix} E_s^+ \\ n_s E_s^+ \end{bmatrix} \tag{24}$$

and a relation between the fields in the incident layer (1),

$$\begin{bmatrix} E_1^+ + E_1^- \\ n_1(E_1^+ - E_1^-) \end{bmatrix} = [M] \begin{bmatrix} E_s^+ \\ n_s E_s^+ \end{bmatrix} \tag{25}$$

Solving this system of equations for $E_1^+$ and $E_1^-$ in terms of $E_s^+$ and the elements of $[M]$ yields the complex reflection coefficient $r^{\mathrm{f}}$, and the complex transmission coefficient $t$;

$$r^{\mathrm{f}} = \frac{E_1^-}{E_1^+} = \frac{an_1 + bn_1n_s - c - dn_s}{an_1 + bn_1n_s + c + dn_s} \tag{26}$$

and

$$t = \frac{E_s^+}{E_1^+} = \frac{2}{an_1 + bn_1n_s - c - dn_s} \tag{27}$$

The power flows are proportional to $|E|^2$. Defining the fluxes relative to an area parallel to the boundary gives

$$R^{\mathrm{f}} = |r^{\mathrm{f}}|^2 \tag{28}$$

and

$$T = \frac{n_s}{n_1} |t|^2 \tag{29}$$

Using the methods described in the previous section, the effects of the substrate can be added by treating the thin-film multilayer as if it were a simple interface between bulk media. Knowledge of the reflectance from the back side of the multilayer ($R^{\mathrm{b}}$) is necessary to account for reflection from the substrate–air interface. Reversing the direction of the beam yields an equation analagous to equation (25),

$$\begin{bmatrix} E_s^+ + E_s^- \\ n_s(E_s^+ - E_s^-) \end{bmatrix} = [M]^{-1} \begin{bmatrix} E_1^- \\ -n_1 E_1^- \end{bmatrix} \tag{30}$$

and paralleling the development of equation (28),

$$R^{\mathrm{b}} = |r^{\mathrm{b}}|^2 = \left|\frac{-an_1 + bn_1n_s - c + dn_s}{an_1 + bn_1n_s + c + dn_s}\right|^2 \tag{31}$$

Obtaining overall reflectances and transmittance is a straightforward application of equations (15)–(18). To calculate absorptance in each layer is considerably more difficult and, fortunately, not necessary. There is virtually zero thermal resistance across thin layers and even across many thick solid layers, like sheets of glass. Heat absorbed in the sublayers of any solid coating/substrate unit can be treated as being uniformly distributed throughout the unit; in other words, the quantity of interest is not the individual absorptions of the sublayers, but the absorption in the combination, which is simply obtained from the total reflectance and transmittance values by conservation of energy.

## GENERAL MULTIPLE LAYER WINDOW

The previous section gives a method for calculating $R$, $T$, and $A$ for a bulk substrate with a thin-film multilayer interference coating. The problem now becomes one of extending the methods described above, which deal with an arbitrary number of layers, to include layers that are described in terms of their overall optical properties ($R$, $T$, and $A$) rather than in terms of optical constants and thicknesses ($n$, $\alpha$, and $t$). Not only does this procedure allow treatment of thin-film coated layers as part of some larger window system, but also it opens the possibility of including any sample whose $R$, $T$ and $A$ properties have been determined experimentally rather than only those derived from the optical constants and thicknesses. These experimental data may be for window components which cannot be derived from first principles, as described in the earlier sections of this paper, because of their failure to meet the conditions of simple geometry or composition. In practice, many of the thin-film coated samples should also be described by $R$, $T$ and $A$ data rather than by relying on published optical constants, since there are large variations in reported values due to differences in deposition methods and process variables.

For the general case shown in Figure 4, a series of window elements, which may themselves be composites, are separated by air layers. Solid elements are numbered consecutively from the incident (left) side. The gross properties of these elements may be measured directly or derived by the methods presented in previous sections of this paper. Here, we shall follow a procedure and notation similar to that of the earlier section entitled, 'Series of Thick Layers'.

In equations (32)–(38), $T_{ij}$ and $R_{ij}$ refer to transmittance through and reflectance from elements $i$ to $j$ as if they were standing alone. In general, a physically asymmetric window element, represented by $R_{i,j}$, will also be optically asymmetric with respect to reflection. Reflection is denoted from the incident or

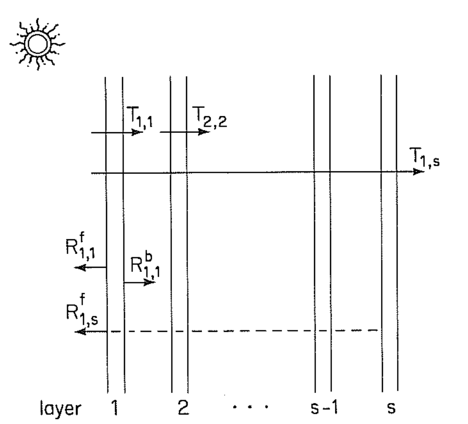


Figure 4. General structure of the layers in a window where the numbered layers are separated by non-absorbing air layers. $T_{i,j}$ is the transmittance through layers $i$ to $j$; $R_{i,j}$ is the reflectance from layers $i$ to $j$

'front' side by an 'f' superscript and from the 'back' side by a 'b' superscript. Recursion relations for $T$ and $R$ for any section of the window can be written in a manner analagous to that presented above. For $i<j$,

$$T_{j,i} = T_{i,j} = \frac{T_{i,j-1}T_{j,j}}{1-R^{\mathrm{f}}_{j,j}R^{\mathrm{b}}_{j-1,i}} \tag{32}$$

$$R^{\mathrm{f}}_{i,j} = R^{\mathrm{f}}_{i,j-1} + \frac{T^2_{i,j-1}R^{\mathrm{f}}_{j,j}}{1-R^{\mathrm{f}}_{j,j}R^{\mathrm{b}}_{j-1,i}} \tag{33}$$

and

$$R^{\mathrm{b}}_{j,i} = R^{\mathrm{b}}_{j,j} + \frac{T^2_{j,j}R^{\mathrm{b}}_{j-1,i}}{1-R^{\mathrm{b}}_{j-1,i}R^{\mathrm{f}}_{j,j}} \tag{34}$$

If the absorption in the $j$th unit standing alone is $A^{\mathrm{f}}_j = 1 - T_{j,j} - R^{\mathrm{f}}_{j,j}$ for radiation incident from the front side, and $A^{\mathrm{b}}_j = 1 - T_j - R^{\mathrm{b}}_{j,j}$ from the back side, then the absorption in the $j$th layer as part of the whole window structure is

$$A_j = T_{1,j-1}A^{\mathrm{f}}_j + \frac{R^{\mathrm{f}}_{j+1,s}T_{j,j}(A^{\mathrm{b}}_j + R^{\mathrm{b}}_{j-1,1}T_{j,j}A^{\mathrm{f}}_j)}{1-T^2_{j,j}R^{\mathrm{f}}_{j+1,s}R^{\mathrm{b}}_{j-1,1}} \tag{35}$$

Three substitutions have been made in equation (33), representing the following subgroups of multiple reflections:

$$T_{1,j-1} = \frac{T_{1,j-1}}{1-R^{\mathrm{f}}_{j,s}R^{\mathrm{b}}_{j-1,1}} \tag{36}$$

$$R^{\mathrm{f}}_{j+1,s} = \frac{R^{\mathrm{f}}_{j+1,s}}{1-R^{\mathrm{f}}_{j+1,s}R^{\mathrm{b}}_{j,1}} \tag{37}$$

and

$$R^{\mathrm{b}}_{j-1,1} = \frac{R^{\mathrm{b}}_{j-1,1}}{1-R^{\mathrm{b}}_{j-1,1}R^{\mathrm{f}}_{j,s}} \tag{38}$$

## RESULTS

The solar optical properties of a variety of window types have been calculated by the methods described in this paper. Results are presented in the form of coefficients similar to those commonly used in detailed building energy analysis programs. The coefficients are produced by performing a least-squares fit of $T(\theta)$, $A_1(\theta)$, $A_2(\theta), \ldots, A_n(\theta)$ to a polynomial in $\cos\theta$. For example, the transmittance of any one of the windows in Table I for any $\theta$ can be calculated from

$$T(\theta) = \sum_{i=0}^{3} c_i \cos^i\theta \tag{39}$$

where the coefficients, $c_0 \cdots c^*$ are provided in Table I. An analogous relation is used for absorptance.

The hemispherical transmittance (or absorptance) is defined by,

$$T_{\mathrm{h}} = 2\int_0^{\pi/2} T(\theta)\cos\theta\sin\theta\,\mathrm{d}\theta \tag{40}$$

Table I. Coefficients for calculating angularly dependent optical properties of windows

| Window* structure | P† | Coefficients | | | | | |
|---|---|---|---|---|---|---|---|
| | | $c_0$ | $c_1$ | $c_2$ | $c_3$ | Normal | Average |
| g | $T$ | −0·0372 | 3·0392 | −3·6360 | 1·4784 | 0·840 | 0·763 |
| | $A_1$ | 0·0738 | 0·2370 | −0·4364 | 0·2168 | 0·089 | 0·100 |
| p | $T$ | −0·0253 | 3·0797 | −3·7509 | 1·5416 | 0·840 | 0·769 |
| | $A_1$ | 0·0474 | 0·0953 | −0·1778 | 0·0878 | 0·052 | 0·057 |
| h | $T$ | −0·0198 | 3·6608 | −4·7940 | 2·0921 | 0·930 | 0·861 |
| | $A_1$ | 0·0679 | 0·1181 | −0·2824 | 0·1497 | 0·052 | 0·065 |
| g–g | $T$ | −0·0898 | 2·2163 | −2·0106 | 0·5879 | 0·709 | 0·617 |
| | $A_1$ | 0·0948 | 0·2610 | −0·5631 | 0·3052 | 0·095 | 0·110 |
| | $A_2$ | 0·0082 | 0·3404 | −0·4766 | 0·2043 | 0·075 | 0·079 |
| g–g–g | $T$ | −0·0916 | 1·6186 | −1·0274 | 0·0915 | 0·600 | 0·510 |
| | $A_1$ | 0·0965 | 0·2994 | −0·6482 | 0·3546 | 0·099 | 0·114 |
| | $A_2$ | 0·0151 | 0·3990 | −0·6276 | 0·2957 | 0·080 | 0·086 |
| | $A_3$ | −0·068 | 0·2818 | −0·3293 | 0·1182 | 0·064 | 0·063 |
| g–p–g | $T$ | −0·0972 | 1·7584 | −1·2787 | 0·2111 | 0·603 | 0·519 |
| | $A_1$ | 0·0991 | 0·3032 | −0·6561 | 0·3592 | 0·101 | 0·117 |
| | $A_2$ | 1·0120 | 0·2083 | −0·3285 | 0·1566 | 0·047 | 0·049 |
| | $A_3$ | −0·0063 | 0·2950 | −0·3574 | 0·1327 | 0·064 | 0·065 |
| g–h–g | $T$ | −0·0950 | 1·7500 | −1·0998 | 0·0967 | 0·661 | 0·559 |
| | $A_1$ | 0·0966 | 0·2862 | −0·6314 | 0·3477 | 0·095 | 0·111 |
| | $A_2$ | 0·0161 | 0·2937 | −0·5169 | 0·2559 | 0·047 | 0·056 |
| | $A_3$ | −0·0059 | 0·3129 | −0·3776 | 0·1410 | 0·070 | 0·070 |
| g–g–g–g | $T$ | −0·0822 | 1·1970 | −0·4293 | −0·1876 | 0·510 | 0·425 |
| | $A_1$ | 0·0965 | 0·3186 | −0·6849 | 0·3752 | 0·101 | 0·117 |
| | $A_2$ | 0·0155 | 0·4330 | −0·6975 | 0·3352 | 0·084 | 0·090 |
| | $A_3$ | −0·0050 | 0·3329 | −0·4423 | 0·1831 | 0·068 | 0·069 |
| | $A_4$ | −0·0100 | 0·2148 | −0·2023 | 0·0518 | 0·054 | 0·052 |
| g–p–p–g | $T$ | −0·0961 | 1·4572 | −0·8678 | 0·0107 | 0·516 | 0·445 |
| | $A_1$ | 0·0999 | 0·3250 | −0·6953 | 0·3811 | 0·106 | 0·122 |
| | $A_2$ | 0·0133 | 0·2307 | −0·3746 | 0·1823 | 0·050 | 0·053 |
| | $A_3$ | −0·0008 | 0·1892 | −0·2577 | 0·1097 | 0·040 | 0·040 |
| | $A_4$ | −0·0106 | 0·2469 | −0·2637 | 0·0816 | 0·055 | 0·055 |
| g–h–h–g | $T$ | −0·0890 | 1·3523 | −0·3967 | −0·2634 | 0·616 | 0·508 |
| | $A_1$ | 0·0967 | 0·3070 | −0·6814 | 0·3771 | 0·096 | 0·112 |
| | $A_2$ | 0·0169 | 0·3126 | −0·5658 | 0·2857 | 0·047 | 0·057 |
| | $A_3$ | −0·0009 | 0·2870 | −0·4451 | 0·2036 | 0·044 | 0·049 |
| | $A_4$ | −0·0103 | 0·2603 | −0·2588 | 0·0738 | 0·066 | 0·063 |

* Abbreviations used for window materials are: g—glass, 1/8-inch, double strength float; p—polyester film, 4 mil; h—antireflected polyester film.

† Solar properties are identified with either a $T$ for transmittance or an $A_i$ for absorptance in layer $i$, counting from the outside.

This integral can be evaluated directly from $T(\theta)$ or by substituting from equation (39) for $T(\theta)$ to obtain $T_h$ in terms of the coefficients $c_i$:

$$T_h = 2 \sum_{i=0}^{3} \frac{c_i}{i+2} \tag{41}$$

The hemispherical average transmittance or absorptance is calculated and included as the last number in each series of coefficients. These coefficients are a complete set of information in a form that facilitates the calculation of solar heat gain.

Coefficients for ordinary glass windows as well as for newer designs incorporating polyester and high-transmittance polyester films instead of glass are listed in Table I. Optical indices and transmittances of window glass and polyester film are widely available in the literature. Measured properties for a particular type of anti-reflected polyester were obtained from Ruth *et al.* (1979). Each layer is separated by a space filled with air or a gas with low thermal conductance. Since these gases are essentially non-absorbing, changing the width of the air-space will not affect the solar properties of the window, although the thermal characteristics may be drastically altered (Rubin *et al.*, 1980).

## CONCLUSIONS

Depending on the application and the input data available, some combination of the procedures in this paper can be used to obtain the optical properties of an arbitrarily complex window system. For some of the clear windows whose properties are given in the previous section, $T$, $R$, and $A$ remain fairly constant for incident angles up to about 45°. This fact has been used to justify using normal transmittance values to calculate solar heat gain; however, the angle of incidence of sunlight on vertical windows is often greater than 45° except in midwinter or in extreme polar latitudes. Furthermore, windows with layers that are not parallel-sided, such as louvred screens do not even approximately obey this rule. Sun angle should be taken into account either by explicitly using angular dependent properties in an hour-by-hour calculation or by using a precalculated time-averaged number for a given location, such as the 'average transmittance factor' introduced by Berman and Silverstein (1975). More work needs to be done with these averaged optical properties. When in doubt, use hemispherical averaged numbers rather than normal directional values.

As expected, the transmittance and absorptance go to zero and the reflectance goes to one as $\theta$ approaches zero. The absorption curve typically has a broad maximum around 65°. This peak occurs because bulk absorption, due to increasing path length, is increasing at a faster rate than reflectance for angles up to 65°.

Results will be slightly different if one starts with the total properties of each layer, as in the section entitled 'General Multiple Layer Window', rather than the optical constants and thicknesses, as described in the preceding two sections. In this section, combining the polarized components of light to obtain average optical properties for each layer is premature. Otherwise the two methods would be equivalent.

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